ATUL VIDYALAYA
SECOND PRELIMINARY EXAMINATION-2012-13
MATHEMATICS

STD:XII Science **MM: 100**
DATE: __/__/_____ **TIME: 3 hrs**

GENERAL INSTRUCTION
(Three hours)

*(Candidate are allowed additional 15 minutes for **only** reading the paper .*
*They must **NOT** start writing during this time)*

*There will be one paper of **three** hours duration of 100 marks. The syllabus is divided into three sections A , B and C. Section A is compulsory for all candidates. Candidates will have choice of attempting questions from **either** from Section B or Section C.*

***Section A(80 marks)** will consists of 9 questions. Candidate will be required to answer **Question -1 (Compulsory)** and **five** out of the rest of the eight question.*

***Section B/C(20 marks)** Candidate will be required to answer two questions out of three from either Section B or Section C.*

Section – A

Question 1.

i) Find k so that the straight line $y = 2x + k$ may touch the ellipse $3x^2 + 5y^2 = 15$. [3]

ii) Find the value of $\cos\left(\sin^{-1}\frac{3}{5} + \tan^{-1}\frac{3}{4}\right)$. [3]

iii) Differentiate $x^{\sqrt{x}}$ with respect to x . [3]

iv) Use Cramer's rule to solve $2x + 3y = 10$ and $x + 6y = 4$. [3]

v) Find $\frac{dy}{dx}$, if $y + \sin y = x^2$. [3]

vi) Evaluate : $\lim_{x\to 0}\frac{1-\cos 2x}{x^2}$ [3]

vii) Verify Rolle's Theorem for the function $f(x) = x(x-3)^2$ in the interval [0 , 3]. [3]

iix) Show that $f(x) = x^3 - 6x^2 + 9x - 8$ has a maximum value at $x = 1$ and a mimimum value at $x = 3$. [3]

ix) Evaluate : $\int \frac{\cos\sqrt{x}}{\sqrt{x}}dx$. [3]

x) Solve : $\frac{dy}{dx} + 1 = e^{x+y}$ [3]

Question 2

i) Solve with the help of determinants:

$$x + y + z = 9$$
$$2x + 5y + 7z = 52$$
$$2x + y - z = 0$$

[5]

ii) If $y = a\cos(\log x) + b\sin(\log x)$, prove that $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$. [5]

Question 3

i) The sum of three numbers is 2 .If twice the second number is added to the the sum of first and third , the sum is 1 . By adding second and third number to five times the first number , we get 6.Find the three numbers by using matrices. [5]

ii) A , B , C represent three switches in the " ON " position and A' , B' , C' represent in the " OFF " position .

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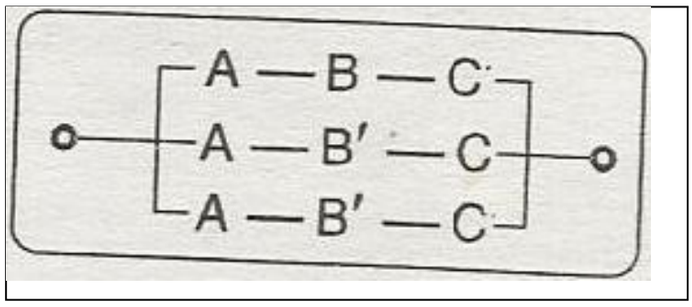

a) Write the polynomial for this switching circuit .

b) Use Boolean algebra to simplify the circuit .

c) Construct the simplified circuit . [5]

Question 4

i) Verify Lagrange's Mean Value Theorem for the given function $f(x)=x(x+3)(x-2)$ on [-1 , 4] [5]

ii) Find the foci and equation of the directrices of the hyperbola $9x^2-16y^2+18x+64y=199$ [5]

Question 5

i) Evaluate : $\int_1^3 \log\left(x+\sqrt{x^2+1}\right)dx$ [5]

ii) Prove that $\cos^{-1}\left(\frac{2+3\cos x}{3+2\cos x}\right)=2\tan^{-1}\left(\frac{1}{\sqrt{5}}\tan\frac{x}{2}\right)$. [5]

Question 6

i) A wire of given length is cut into two portions which are bent into the shape of a circle and a square respectively .Show that the sum of the areas of the circle and the square will be least when the side of the square is equal to the diameter of the circle. [5]

ii) Ten students got the following percentage of marks in Mathematics and Physics:

Marks in Mathematics	56	64	75	85	85	87	91	95	97	98
Marks in Physics	66	72	56	66	74	78	74	88	90	89

Calculate Spearman's coefficient of rank correlation and comment on r . [5]

Question 7

i) Calculate the area of the region bounded by the curve $y=2x-x^2$ and the line $y=x$. [5]

ii) For the data given below , find the regression equation of X on Y .Using the equation calculate the value of X when Y = 15 .

X	20	25	30	35	40	45

Y	12	14	16	20	22	25

Question 8

i) The letters of the word STATISTICS are written on ten identical cards .If two cards are drawn at random, what is the probability that one " S " and one " I " will occur ? [5]

ii) A bag X contains 3 white balls and 2 black balls ; and another bag Y contains 2 white balls and 4 black balls .A bag and a ball out of it are picked at random. What is the probability that the ball is white ? [5]

Question 9

i) Solve : $\left(1+y+x^2y\right)dx+\left(x+x^3\right)dy=0$ [2]

ii) Solve : $\sin^{-1}\left(\frac{dy}{dx}\right)=x+y$ [2]

iii) If 1 , ω , ω^2 are the three cube roots of unity , show that

$$\left(x+y\right)^2+\left(x\omega+y\omega^2\right)^2+\left(x\omega^2+y\omega\right)^2=6xy$$ [4]

iv) Locate the points representing the complex numbers Z on the Argand diagram $\left|Z-i\right|\langle 1$. [2]

(Answer *two* questions from *either* Section B or Section C)

SECTION B

Question 10

i) Find the area of the parallelogram whose adjacent sides are determined by the Vectors $a=\hat{i}+2\hat{j}+3\hat{k}$, $b=3\hat{i}-2\hat{j}+\hat{k}$. [5]

ii) Show that $\left[a+b,b+c,c+a\right]=2\left[abc\right]$ [5]

Question 11

i) Find the equations of the lines passing through the point (-11 , 3 , -2) and perpendicular to two lines $\frac{x}{1}=\frac{y}{2}=\frac{z}{3}$ and $\frac{x+2}{-3}=\frac{y-1}{2}=\frac{z+1}{5}$. [5]

ii) Find the equation of the plane which contains the lines $\frac{x-1}{2}=\frac{y+1}{-1}=\frac{z-3}{4}$ and is perpendiculars to the plane $x+2y+z=12$. [5]

Question 12

i) Suppose 5 men out of 100 and 25 women out of 1000 are good orators . An orator is chosen at random. Find the probability that a male person is selected. Assume that there are equal number of men & women . [5]

ii) A random variable X has the following probability distribution:

$X\left(=xi\right)$	**2**	**5**	**6**	**7**
$P\left(X=xi\right)$	$\frac{1}{10}$	K	$\frac{3}{10}$	$\frac{4}{10}$

Find mean and variance of X. [5]

SECTION C

Question 13

i) Solve the following linear programming problem graphically : [5]

Maximize $Z = 60x + 15y$

Subject to constraints

$$x + y \le 50$$
$$3x + y \le 90$$
$$x, y \ge 0$$

ii) The true discount on a bill 9 month hence at 6% per annum is ` 360 . Find the amount of the bill and its present worth. [5]

Question 14

i) A sum of ` 2522 is borrowed from a money lender at 5% per annum compounded annually . If this amount is to be paid back in 3 equal installments , find the annual installments . [5]

ii) A firm's marginal revenue is $MR = 20e^{-\frac{x}{10}}\left(1 - \frac{x}{10}\right)$ .Find the corresponding demand function. [5]

Question 15

i) The following are the group index numbers and the group weights of an average working class family budget .

Construct the cost of living index number assigning given weights . [5]

Group	Weight	Index Number
Food	43	352
Fuel & lighting	10	220
Clothing	8	230
House Rent	12	160
Miscellaneous	22	190

ii) Compute four yearly centred moving averages for the following series of Observation (values upto three place of decimal). [5]

Year	1985	1986	1987	1988	1989	1990	1991	1992
Annual Sales	3.6	4.3	4.3	3.4	4.4	5.5	3.4	2.4